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On the one-loop Kähler potential in five-dimensional brane-world supergravity

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Abstract: We present an on-shell formulation of 5d gauged supergravity coupled to chiral matter multiplets localized at the orbifold fixed points. The brane action is constructed via the Noether method. In such set-up we compute one-loop corrections to the Kähler potential of the effective 4d supergravity and compare the result with previous computations based on the off-shell formalism. The results agree at lowest order in brane sources, however at higher order there are differences. We explain this discrepancy by an ambiguity in resolving singularities associated with the presence of infinitely thin branes.

Keywords: Field Theories in Higher Dimensions, Supersymmetry Breaking, Supergravity Model.

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1. Introduction

Supersymmetry breaking and its mediation to the observable sector is one of the most important problems in physics beyond the Standard Model. An acceptable theory of supersymmetry breaking is strongly constrained by the observed features of the low-energy physics. Spontaneous breaking must occur in a hidden sector and must be transmitted to the observable sector via non-renormalizable operators. Gravity mediation is an attractive and economical possibility, but it is well known that generic models face the flavor problem.

It has been noted in ref. [1] that a spatial separation of the hidden and observable sectors brings new elements into the mechanism of gravity mediation. The simplest set-up that provides for such sequestering is that of five-dimensional (5d) supergravity compactified on an orbifold in which the chiral matter of the observable and hidden sectors is localized on the two different boundaries of the fifth dimension. In the minimal set-up with no matter fields in the bulk the tree-level Kähler potential of the effective 4d theory has a special structure that results in absence of tree-level transmission of supersymmetry breaking. Supersymmetry breaking is then transmitted to the observable sector at one-loop level by (flavor-blind) gravitational interactions. One mechanism that can operate here is anomaly mediation [1, 2]. Besides, there is always a contribution from integrating out the Kaluza-Klein (KK) tower of the supergravity multiplet. Its effect on the low-energy phenomenology can be summarized as a correction to the tree-level Kähler potential of the 4d effective supergravity. This one-loop correction was computed in refs. [3 – 6]. Unlike in four-dimensions, the contact terms between the hidden and the observable sectors generated by gravity loops are finite and calculable. Therefore 5d supergravity models open a possibility of constructing a realistic and predictive theory of soft terms. See also [7] for other studies of 5d brane-world supergravity.

Given the important role of gravitational loop corrections it is advantageous to study them in a somewhat different setting. The brane-world action considered in refs. [3 – 6] was based on an off-shell formulation of 5d supergravity. In this paper we point out that the physics of 5d brane-worlds can be studied in a simpler set-up of on-shell supergravity. Using the Noether procedure, we construct a locally supersymmetric action for an $N = 1$ chiral multiplet confined to a 4d brane and coupled to 5d gauged supergravity in the bulk. In such set-up we compute the one-loop corrections to the Kähler potential and compare it with the previous results.

In principle, the Noether method is less powerful than the off-shell formalism of ref. [8] or the superconformal tensor calculus of ref. [9]. Still, we will argue that it has several advantages. Firstly, it is obviously less involved. The number of fields is reduced and no advanced superspace techniques are needed for constructing the action. We will also see that one-loop computations are considerably simplified in this set-up. Secondly, it facilitates the treatment of singularities associated with the presence of infinitely thin (delta-like) branes. In the off-shell formulation integrating out auxiliary fields generates singular terms in the brane action. These singular terms can be avoided in our Noether formulation. Furthermore, working in our set-up we will notice certain ambiguity in defining the brane-world action that is connected to arbitrariness in resolving the singular behaviour of bulk fields near the branes. In certain circumstances, namely when brane sources are large compared to the compactification scale, this ambiguity may also affect low-energy observables. Finally, the procedure can be readily generalized to higher-dimensional spacetimes where an off-shell formulation of supergravity does not exist (for example, similar method was used for coupling 10d brane to 11d supergravity in the Horava-Witten model [10]).

The paper is organized as follows. In section 2 we construct an on-shell action for an $N = 1$ chiral multiplet coupled to 5d supergravity. In section 3 we derive the tree-level Kähler potential describing the dynamics of the low energy degrees of freedom in this set-up. In section 4 we compute the one-loop correction to the Kähler potential and in section 5 we comment on the differences with the previous works. In this paper we restrict to studying technical issues associated with the Noether construction and one-loop computation. Phenomenological issues, like moduli stabilization or determination of soft breaking terms, are left for future publications.

2. Five-dimensional brane-world action

In this section we construct a locally supersymmetric action for an $N = 1$ chiral multiplet confined to a 4d brane and coupled to 5d supergravity in the bulk. We use the Noether method. That is, starting with a globally supersymmetric action for the brane multiplet we systematically add new terms to the action and supersymmetry transformations until the bulk+brane set-up becomes locally supersymmetric. We first work out all necessary zero-

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and two-fermion terms such that all two-fermion supersymmetric variations of the brane action cancel. The next step is to determine four-fermion terms from cancellation of four-fermion variations. In fact, the latter step will not be presented here, but see [11]. It should be stressed however, that at the two-fermion level the brane action we construct is locally supersymmetric to arbitrary power in $1/M_5$ expansion, where M_5 is the 5d Planck scale.

The 5d bulk contains $N = 2$ supergravity multiplet[1] $(e^a_\alpha, \psi_\alpha, \mathcal{A}_\alpha)$. For the flat (ungauged) 5d supergravity the action up to four-fermion terms reads [12]

$$\mathcal{L} = M_5^3 e_5 \bigg[\frac{1}{2} R_5 - i\overline{\psi_\alpha}\Gamma^{\alpha\beta\gamma} D_\beta\psi_\gamma - \frac{1}{4}\mathcal{F}_{\alpha\beta}\mathcal{F}^{\alpha\beta} + \frac{1}{6\sqrt{6}}\epsilon^{\alpha\beta\gamma\delta\epsilon}\mathcal{A}_\alpha\mathcal{F}_{\beta\gamma}\mathcal{F}_{\delta\epsilon} + \\ + \frac{3i}{4\sqrt{6}}\overline{\psi_\alpha}\Gamma^{\alpha\beta\gamma\delta}\psi_\beta\mathcal{F}_{\gamma\delta} + \frac{3i}{2\sqrt{6}}\overline{\psi_\alpha}\psi_\beta\mathcal{F}^{\alpha\beta}\bigg], \tag{2.1}$$

while the supersymmetry transformations, up to three-fermion terms in $\delta\psi$ are given by

$$\begin{aligned} \delta e^a_\alpha &= \frac{i}{2}\overline{\psi_\alpha}\Gamma^a\epsilon + \text{h.c.}\,, \\ \delta\psi_\alpha &= D_\alpha\epsilon - \frac{1}{4\sqrt{6}}(\Gamma_{\alpha\beta\gamma} - 4g_{\alpha\beta}\Gamma_\gamma)\epsilon\mathcal{F}^{\beta\gamma}\,, \\ \delta\mathcal{A}_\alpha &= -\frac{i\sqrt{6}}{4}\overline{\psi_\alpha}\epsilon + \text{h.c.}\,. \end{aligned} \tag{2.2}$$

The fifth dimension is the orbifold S_1/Z_2 parametrized by $x_5 \in [-\pi R, \pi R]$ with Z_2 acting as $x_5 \to -x_5$. Under Z_2 the field components e^m_μ, e^5_5, $\mathcal{A}_5$, $\psi^+_\mu \equiv P_R\psi_\mu$ and $\psi^+_5 \equiv P_L\psi_5$ are even, $\psi(-x_5) = \psi(x_5)$, while e^5_μ, e^m_5, $\mathcal{A}_\mu$, $\psi^-_\mu \equiv P_L\psi_\mu$ and $\psi^-_5 \equiv P_R\psi_5$ are odd, $\psi(-x_5) = -\psi(x_5)$. At the orbifold fixed point $x_5 = 0$ we couple the $N = 1$ chiral multiplet $[Q_0, \psi_Q]$ $(P_L\psi_Q = \psi_Q)$. Of course the action for the matter $[Q_\pi, \psi_{Q_\pi}]$ at the orbifold fixed point $x_5 = \pi R$ can be constructed analogously.

The starting point for the Noether method is the action

$$\mathcal{L}_1 = e_4\delta(x_5)\left\{\partial_\mu Q_0^\dagger\partial^\mu Q_0 + i\overline{\psi}_Q\gamma^\mu D_\mu\psi_Q\right\} \tag{2.3}$$

invariant under global supersymmetry transformations

$$\delta Q_0 = \frac{1}{\sqrt{2}}\bar{\epsilon}\psi_Q \qquad \delta\psi_Q = -\frac{1}{\sqrt{2}}i\gamma^\mu\partial_\mu Q_0\,\epsilon\,. \tag{2.4}$$

The boundary lagrangian (2.3) is clearly not the most general starting point. One could consider several chiral multiplets with arbitrary sigma-model kinetic terms. In such case the Noether procedure would be more tedious but qualitatively similar and our results obtained in the minimal case can be straightforwardly generalized. A more non-trivial modification of eq. (2.3) is to include gravity brane kinetic terms, that is $\Delta\mathcal{L} = e_4\delta(x_5)\Omega(Q_0)R_4$ where R_4 is the Ricci scalar of the 4d metric induced at the boundary and $\Omega(Q_0)$ is a real funtion of the brane scalars. In such case application of the Noether procedure leads to appearance of singular $\delta(x_5)^n$ terms, which signals ambiguity in defining the brane action. We postpone the discussion of this case untill section 5.

[1]Our notation and conventions are summarized in A.

Under the transformations (2.4) but with ϵ depending on the 4d coordinates x_μ the lagrangian of eq. (2.3) transforms as $\delta\mathcal{L} = \partial_\mu\epsilon\, j^\mu$, where j^μ is the Noether current of global supersymmetry (the supercurrent). In order to cancel this variation we need to couple one linear combination ψ_μ of the two bulk gravitinos $\psi^+_\mu(0)$, $C\overline{\psi^-_\mu}^T(0)$ to the the supercurrent and identify the parameter ϵ with the corresponding combination of the two bulk supersymmetry transformation parameters, $\epsilon_+(0)$ and $C\overline{\epsilon_-}^T(0)$. In absence of brane sources for the gravitinos $\psi^-_\mu(0) = 0$ and thus we choose $\psi_\mu = \psi^+_\mu(0)$. However, if such sources are present some other combination of the gravitinos couples to the brane matter. This is for example the case when gravitino brane mass terms are present, see the discussion in B.

Thus we add to the brane action the so-called Noether term,

$$\mathcal{L}_2 = -\frac{1}{\sqrt{2}} e_4 \delta(x_5) \overline{\psi}_Q \gamma^\mu \gamma^\rho \partial_\rho Q_0 \psi_\mu + \text{h.c.}\,. \tag{2.5}$$

At the level of two-fermion terms there are still variations to be canceled. One originates from varying the gravitino in eq. (2.5), $\delta\psi_\mu \sim \mathcal{F}_{\mu 5}$, the other from variation of the vielbein in the kinetic terms of eq. (2.3). It turns out that the necessary modifications of the brane action can be concisely summarized as the redefinition of the graviphoton field strength. Namely, in the 5d bulk action (2.1) and supersymmetry transformations (2.2) we replace $\mathcal{F}_{\mu 5}$ with $\hat{\mathcal{F}}_{\mu 5}$ defined as

$$\begin{aligned} \hat{\mathcal{F}}_{\mu 5} &= \mathcal{F}_{\mu 5} + \frac{1}{M_5^3}\delta(x_5) j^0_\mu\,, \\ j^0_\mu &= \frac{i}{\sqrt{6}}\left[Q_0^\dagger \partial_\mu Q_0 - \partial_\mu Q_0^\dagger Q_0 + \frac{i}{2}\overline{\psi}_Q \gamma_\mu \psi_Q\right], \end{aligned} \tag{2.6}$$

and modify the transformation law of the graviphoton by

$$\delta\mathcal{A}_5 = \frac{i}{\sqrt{12}}\delta(x_5)\overline{\psi}_Q \epsilon Q_0 + \text{h.c.} \tag{2.7}$$

In other words we modify the Bianchi identity for the graviphoton field strength such that $\partial_{[\mu}\hat{\mathcal{F}}_{\nu]\,5} = \frac{2i}{\sqrt{6}M_5^3}\delta(x_5)\partial_{[\mu}Q_0^\dagger\partial_{\nu]}Q_0$. The replacement $\mathcal{F} \to \hat{\mathcal{F}}$ generates singular δ^2 terms in the brane action. However such singular terms are absent in the low energy effective theory after integrating out the graviphoton $\mathcal{A}_\mu$. The reason for this is precisely the fact that the singular δ^2 terms match the full square structure inside the graviphoton field strength. In the 5d setup these singular terms provide for necessary counterterms to cancel divergences in certain one-loop diagrams [13]. No other singular terms arise in this construction.[2]

The on-shell action we derived by the Noether method differs from the brane action obtained in the off-shell formalism after eliminating the auxiliary fields [5]. In particular the kinetic terms of the gravity multiplet do not couple to the brane here. In section 5 we will discuss this issue more carefully and argue that the two formalisms are related by a redefinition of the 5d degrees of freedom. Note that the choice of variables we use here is very convenient, as no singular δ^n terms are present in the on-shell action (except for the δ^2 fitting the full square).

[2] The Noether construction of brane action in flat 5d supergravity was also pursued in ref. [14] but their results differ from ours, notably by the absence of the full square structure.

At this point all the two-fermion variations are canceled. More involved calculations are needed to work out four-fermion terms in the brane action as well as three-fermion modifications of the supersymmetry transformation laws. They are not necessary for the following analysis and will not be presented here.[3] Indeed, we shall see that the form of the low energy 4d supergravity (including one-loop corrections) can be read out from the terms we have already derived. Note also that once tree-level effective supergravity is known all the three- and four-fermion terms can be easily inferred by matching with the canonical 4d supergravity lagrangian.

This construction of the brane world action can be carried over to the case of warped supergravity, that is, 5d supergravity with a U(1) subgroup of the SU(2) R-symmetry group gauged by the graviphoton [15]. The 5d action can be obtained from the flat one in eq. (2.1) by replacing all the derivatives acting on the gravitino by

$$D_\alpha \psi_\beta \to D_\alpha \psi_\beta + \frac{i}{2} k\epsilon(x_5)\Gamma_\alpha \psi_\beta + \frac{i\sqrt{6}}{2} k\epsilon(x_5)\mathcal{A}_\alpha \psi_\beta \,. \tag{2.8}$$

Analogous replacement should be done for $D_\alpha \epsilon$ in the gravitino transformation laws. Besides, the 5d bulk action(2.1) should be supplemented by

$$\mathcal{L} = 6M_5^3 k^2 e_5 \,. \tag{2.9}$$

Hence the gauging implies the presence of a negative cosmological constant in the bulk and so the gravitational background solution is AdS_5. On the orbifold, the presence of the step function $\epsilon(x_5)$ induces additional variations proportional to the delta function,

$$\delta\mathcal{L} = M_5^3 e_4 \delta(x_5) k \left[3i\overline{\psi_\mu}\gamma^\mu\gamma^5\epsilon - \sqrt{6}i\overline{\psi_\mu}\gamma^{\mu\nu}\gamma^5\epsilon\mathcal{A}_\nu\right](\delta(x_5) - \delta(x_5 - \pi R)) \,. \tag{2.10}$$

Canceling the first term requires the presence of the brane tension [16],

$$\mathcal{L} = -6M_5^3 e_5 k(\delta(x_5) - \delta(x_5 - \pi R)) \,. \tag{2.11}$$

and so the gravitational background in this set-up is precisely that of the Randall-Sundrum model [17]. In absence of brane matter the second term vanishes. When brane chiral multiplets are present the current j^0_μ in eq. (2.6) acts as a source for the graviphoton so that it has a jump at the brane, $\mathcal{A}_\mu \sim \epsilon(x_5)\frac{1}{2M_5^3} j^0_\mu$. Moreover, the Noether term (2.5) is a source for the negative parity gravitino. By equations of motion it behaves as $\gamma^{\mu\nu}\psi^-_\nu \sim \frac{1}{2\sqrt{2}}\epsilon(x_5)\gamma^\rho\gamma^\mu\psi_Q\partial_\rho Q_0$ near the brane. In the flat case $k \to 0$ these subtleties in boundary conditions do not affect the Noether construction at the level of two-fermion terms. But for $k \neq 0$ these boundaries conditions imply that the second term in eq. (2.10) is non-vanishing.[4] It turns out that the necessary modification that cancels this term consists in multiplying the brane action by a $|Q_0|^2$ dependent factor,

$$\omega_0(|Q_0|^2) = \frac{1}{1 - \frac{k|Q_0|^2}{3M_5^3}} \,. \tag{2.12}$$

[3] The complete action is given in ref. [11]. See also this reference for coupling of $N = 1$ gauge multiplets on the brane.

[4] A similar treatment of the boundary conditions in supersymmetric variations was also employed in refs. [18, 19].

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The brane-world action up to four-fermion terms reads

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_{\rm bulk}(\mathcal{F}_{\mu 5} \to \mathcal{F}_{\mu 5} + \frac{1}{M_5^3}\delta(x_5) j^0_\mu) + \delta(x_5)\mathcal{L}_{\rm brane}\,,\\
j^0_\mu &= \frac{i}{\sqrt{6}}\omega_0\left(Q_0^\dagger \partial_\mu Q_0 - \partial_\mu Q_0^\dagger Q_0\right) - \frac{1}{2\sqrt{6}}\omega_0^2 \psi_Q \gamma_\mu \psi_Q\,,\\
\mathcal{L}_{\rm brane} &= -6M_5^3 k e_5 + e_4 \omega_0^2 \times \\
&\quad \times \left[\frac{1}{2}\partial_\mu Q_0^\dagger \partial^\mu Q_0 + \frac{i}{2}\overline{\psi}_Q \gamma^\mu D_\mu \psi_Q - \frac{1}{\sqrt{2}} e_4 \delta(x_5) \overline{\psi}_Q \gamma^\mu \gamma^\rho \partial_\rho Q_0 \psi_\mu + \text{h.c.}\right].
\end{aligned}
\tag{2.13}
$$

One should also insert the factor ω_0 into the transformation of the graviphoton in eq. (2.7). The action for the matter on the brane at $x_5 = \pi R$ is analogous with $\delta(x_5) \to \delta(x_5 - \pi R)$, $k \to -k$, $Q_0 \to Q_\pi$, $\psi_Q \to \psi_{Q_\pi}$.

One can further extend the model to include a non-trivial superpotential $W_0(Q_0)$ for the brane multiplet. The treatment of the boundary conditions is then much more involved and in this paper we only discuss some limiting cases. However this discussion is not necessary for our computation of one-loop corrections and so we shift it to B.

3. Four-dimensional effective supergravity

We move to discussing the form of the 4d effective supergravity that describes the light degrees of freedom (those with masses below the compactification scale) of the 5d theory compactified on the background

$$
ds^2 = a^2(x_5) g_{\mu\nu} dx^\mu dx^\nu - \phi^2 dx_5^2 \qquad a(x_5) = e^{-k\phi x_5}\,. \tag{3.1}
$$

The limit $k \to 0$ corresponds to flat compactification. The bosonic degrees of freedom are the 4d metric $g_{\mu\nu}$, the radion $\phi \equiv \sqrt{g_{55}}$, the fifth component of the graviphoton $\mathcal{A}_5$ and the scalars on the two branes Q_0 and Q_π. The Kähler potential of the 4d supergravity can be found by matching with the kinetic terms for those fields.

The kinetic terms for the metric component are obtained by inserting the background (3.1) into the 5d Einstein-Hilbert action,

$$
\mathcal{L} = \sqrt{-g} M_p^2 \left[\frac{1}{2}\frac{1-a_\pi^2}{2k\pi R} R(g) + \frac{3}{2} k\pi R\, a_\pi^2 (\partial_\mu \phi)^2\right], \tag{3.2}
$$

where $a_\pi = e^{-k\pi R\phi}$ and $M_p^2 = 2\pi R M_5^3$. To go the Einstein basis we need to perform the Weyl rescaling $g_{\mu\nu} \to \frac{1}{f_E(\phi)} g^{(E)}_{\mu\nu}$ with $f_E(\phi) = \frac{1-a_\pi^2}{2k\pi R}$. Then the kinetic terms become

$$
\mathcal{L} = \sqrt{-g^{(E)}} M_p^2 \left[\frac{1}{2} R(g) + \frac{3}{4 f_E(\phi)^2} a_\pi^2 (\partial_\mu \phi)^2\right]. \tag{3.3}
$$

The kinetic terms in eq. (2.13) yield

$$
\mathcal{L} = \sqrt{-g^{(E)}} \frac{1}{f_E(\phi)} \left[\omega_0^2 \partial_\mu Q_0^\dagger \partial^\mu Q_0 + a_\pi^2\, \omega_\pi^2\, \partial_\mu Q_\pi^\dagger \partial^\mu Q_\pi\right]. \tag{3.4}
$$

More care is needed to derive kinetic terms of $\mathcal{A}_5$. To do this consistently we need to integrate out the negative parity components of the graviphoton $\mathcal{A}_\mu$. The relevant part of the 5d action is

$$\mathcal{L}_{5d} = \frac{1}{2\phi} a^2(x_5)\sqrt{-g} M_5^3 \left(\partial_5 \mathcal{A}_\mu - \partial_\mu \mathcal{A}_5 + \frac{1}{M_5^3}\delta(x_5) j_\mu^0 + \frac{1}{M_5^3}\delta(x_5 - \pi R) j_\mu^\pi\right)^2 . \tag{3.5}$$

The solution to the graviphoton equations of motion is $\mathcal{A}_\mu = \partial_\mu \mathcal{A}_5 x_5 + \frac{1}{2k}\epsilon(x_5) C_\mu a^{-2}(x_5) + \epsilon(x_5) D_\mu$. The boundary conditions $\mathcal{A}_\mu(0) = j_\mu^0/2$, $\mathcal{A}_\mu(\pi R) = -j_\mu^\pi/2$ determine the integration constants and we find $C_\mu = -a_\pi^2 \frac{\partial_\mu \mathcal{A}_5 + (j_\mu^0 + j_\mu^\pi)/M_p^2}{f_E(\phi)}$. Inserting this solution back into the 5d action and integrating over x_5 yields

$$\mathcal{L}_4 = \frac{1}{2} M_p^2 \sqrt{-g^{(E)}} \frac{a_\pi^2}{f_E(\phi)^2} \left(\partial_\mu \mathcal{A}_5 + \frac{1}{M_p^2}(j_\mu^0 + j_\mu^\pi)\right)^2 . \tag{3.6}$$

Note that the δ^2 terms has canceled. The Kähler potential that reproduces the kinetic terms (3.3), (3.4) and (3.6) is given by $K = -3\log\Omega$ where

$$\begin{aligned}
\Omega &= \frac{1 - e^{-k\pi R(T+\overline{T})}}{2k\pi R} - \frac{1}{3M_p^2}|Q_0|^2 - \frac{1}{3M_p^2} e^{-k\pi R(T+\overline{T})}|Q_\pi|^2 , \\
\mathrm{Re}\, T &= \phi - \frac{1}{2k\pi R}\log\left(1 - \frac{2k\pi R}{3M_p^2}|Q_0|^2\right) + \frac{1}{2k\pi R}\log\left(1 + \frac{2k\pi R}{3M_p^2}|Q_\pi|^2\right) , \\
\mathrm{Im}\, T &= i\sqrt{\frac{2}{3}}\mathcal{A}_5 .
\end{aligned} \tag{3.7}$$

One can check that also the remaining interaction terms in the brane-world action (2.13) fit the general structure of 4d supergravity [20] with the Kähler potential of eq. (3.7). Generalization to an arbitrary number of brane matter multiplets with general kinetic terms is straightforward. It amounts to replacing $|Q_i|^2$ with arbitrary real functions $\Omega_i(Q_i^n)$

$$\begin{aligned}
\Omega &= \frac{1 - e^{-k\pi R(T+\overline{T})}}{2k\pi R} - \frac{1}{3M_p^2}\Omega_0(Q_0^n) - \frac{1}{3M_p^2} e^{-k\pi R(T+\overline{T})}\Omega_\pi(Q_\pi^m) , \\
\mathrm{Re}\, T &= \phi - \frac{1}{2k\pi R}\log\left(1 - \frac{2k\pi R}{3M_p^2}\Omega_0(Q_0^n)\right) + \frac{1}{2k\pi R}\log\left(1 + \frac{2k\pi R}{3M_p^2}\Omega_\pi(Q_\pi^m)\right) .
\end{aligned} \tag{3.8}$$

In the flat limit $k \to 0$ we recover the well-known no-scale structure,

$$\begin{aligned}
\Omega &= \frac{T + \overline{T}}{2} - \frac{1}{3M_p^2}\Omega_0(Q_0^n) - \frac{1}{3M_p^2}\Omega_\pi(Q_\pi^m) , \\
T &= \phi + \frac{1}{3M_p^2}\Omega_0(Q_0^n) + \frac{1}{3M_p^2}\Omega_\pi(Q_\pi^m) + i\sqrt{\frac{2}{3}}\mathcal{A}_5 .
\end{aligned} \tag{3.9}$$

Furthermore, in the presence of brane superpotential $W_0(Q_0^i)$ and $W_\pi(Q_\pi^i)$ the superpotential of the effective 4d supergravity reads

$$W = W_0(Q_0^i) + e^{-3k\pi RT} W_\pi(Q_\pi^i) . \tag{3.10}$$

The Kähler potential derived here is the same function of T and Q as the one in ref. [6] (note that we use the definition of Ω that differs by a factor $-1/(3M_p^2)$ from that of ref [6]). However the definition of the modulus T in terms of the 5d degrees of freedom is different (in our formulation it is also a function the brane matter fields). Of course, at tree-level the physics (like moduli stabilization, transmission of supersymmetry breaking) is the same in both formalisms. In particular the Kähler potential in eq. (3.8) implies no tree-level mediation of supersymmetry breaking through the bulk (although for $k \neq 0$ it is not of the no-scale form).

4. One-loop corrections to the Kähler potential

We now use our on-shell formulation of the 5d theory to compute one-loop corrections to the tree-level Kähler potential. From the point of view of the 4d effective theory no symmetry protects the particular structure of Ω in eq. (3.8). We expect that $\Omega_{1\text{ loop}} = \Omega + \Delta\Omega$ and that $\Delta\Omega$ includes couplings other than those in eq. (3.8), for example higher powers of $e^{-k\pi R(T+\overline{T})}$ or contact terms between Q_0 and Q_π. These new terms will lead to mediation of supersymmetry breaking.

We first compute the one-loop effective action in the full 5d theory and then match to 4d effective supergravity with a Kähler potential $\Omega + \Delta\Omega$. The computation involves regularization of divergent expressions so we first discuss the most general structure of the counterterms in the Kähler potential. Since $\Omega = e^{-K/3}$ is the coefficient of the Einstein-Hilbert term in the supergravity conformal frame,

$$\mathcal{L}_C = \sqrt{-g^C} M_p^2 \left[\frac{1}{2}\Omega R - 3\Omega_{\overline{m}n}\partial_\mu z^\dagger_{\overline{m}}\partial_\mu z_n - \frac{3}{4\Omega}(\Omega_{\overline{m}}\partial_\mu z^\dagger_{\overline{m}} - \Omega_n\partial_\mu z_n)^2 + \cdots\right], \tag{4.1}$$

the possible counterterms are constrained by the most general form of the Einstein-Hilbert terms consistent with 5d general coordinate invariance and locality,

$$\mathcal{L} = C_B\sqrt{-g_5}R_5 + C_0(Q_0)\delta(x_5)\sqrt{-g_4}R_4 + C_\pi(Q_\pi)\delta(x_5 - \pi R)\sqrt{-g_4}R_4\,. \tag{4.2}$$

After compactification on the warped background eq. (3.1) and Weyl rescaling to the conformal frame $g_{\mu\nu} \to f_C g^C_{\mu\nu}$ this becomes

$$\mathcal{L} = \left[C_B\frac{1-a_\pi^2}{2k\pi R} + C_0(Q_0) + C_\pi(Q_\pi)a_\pi^2\right] f_C\sqrt{-g^C}R\,. \tag{4.3}$$

In our case $f_C = 1 - \frac{2k\pi R}{3M_p^2}\Omega_0(Q_0)$. Using eq. (3.8) we express ϕ in $a_\pi = e^{-k\pi R\phi}$ by T and Q and we obtain

$$\begin{aligned}\mathcal{L} = \Bigg[&\left(\frac{1}{2k\pi R}C_B + C_0(Q_0)\right)\left(1 - \frac{2k\pi R}{3M_p^2}\Omega_0(Q_0)\right) + \\ &+ e^{-k\pi R(T+\overline{T})}\left(-\frac{1}{2k\pi R}C_B + C_\pi(Q_\pi)\right)\left(1 + \frac{2k\pi R}{3M_p^2}\Omega_\pi(Q_\pi)\right)\Bigg]\sqrt{-g^C}R\,.\end{aligned} \tag{4.4}$$

We see that the coefficient of the Einstein-Hilbert term is of the same form as the Kähler potential (3.8). We are thus guaranteed that all divergences we encounter in the one-loop computation can be absorbed by renormalization of the parameters in the tree-level

Kähler potential (3.8). In particular, these divergences are not relevant for the questions of supersymmetry breaking mediation through the bulk. On the other hand, any couplings in $\Delta\Omega$ that are different than those in the tree-level Kähler potential correspond necessarily to non-local operators in 5d theory and therefore they should be finite and UV insensitive.

Reference [6] derives a very useful expression for $\Delta\Omega$,

$$\Delta\Omega \sim \int \frac{d^4k}{(2\pi)^4} \sum_n \frac{1}{k^2} \log(k^2 + m_n^2)\,. \tag{4.5}$$

Before we compute $\Delta\Omega$ in our set-up we first present a simple derivation of eq. (4.5). In order to compute $\Delta\Omega$ it is sufficient to compute corrections to the Einstein-Hilbert term in the conformal frame and compare the result with eq. (4.1). Quite generally, a field of spin j and mass m contributes to one-loop renormalization of the Einstein-Hilbert term (in dimensional regularization):

$$\Delta\mathcal{L}_j = n_j \frac{\Gamma(1-d/2)m^{d-2}}{(4\pi)^{d/2}} \sqrt{-g}R\,, \tag{4.6}$$

where $n_0 = 0$ for a conformally coupled scalar, $n_{1/2} = 1/6$ for a Dirac fermion, $n_1 = -1/3$ for a gauge boson, and $n_{3/2} + n_2 = 0$ for a summed contribution of a Dirac gravitino and a graviton. Specializing to the case of 5d sugra, a 5d hypermultiplet contains one Dirac fermion, a 5d vector multiplet - one Dirac fermion and one gauge boson, while the gravity multiplet contains one gauge boson, one Dirac gravitino and one graviton at each KK level. Summing all these contributions we find:

$$\Delta\Omega = \frac{1}{3}(-2 - N_V + N_H)\frac{\Gamma(1-d/2)}{M_p^2(4\pi)^{d/2}} \sum_n m_n^{d-2}\,. \tag{4.7}$$

where N_V and N_H is the number of vector multiplets and hypermultiplets, respectively, and m_n are the masses of the KK modes in the conformal frame as a function of background values of T and Q. For $N_V = N_H = 0$ this formula is equivalent to that in ref. [6] with the momentum integral evaluated using dimensional regularization.

We now apply the general formula (4.7) to the model considered in this paper. In the frame set by $g_{\mu\nu}$ in eq. (3.1) the KK spectrum is given by positive roots of the equation:

$$J_1\left(\frac{m_n}{k}\right) Y_1\left(\frac{m_n}{ka_\pi}\right) - Y_1\left(\frac{m_n}{k}\right) J_1\left(\frac{m_n}{ka_\pi}\right) = 0\,. \tag{4.8}$$

Note that in our set-up the KK spectrum m_n is a function of the field ϕ only and is not modified by the presence of brane matter fields Q. Going to the conformal frame $m_n \to m_n f_C^{1/2}$, so $\Delta\Omega$ picks up an additional multiplicative factor $f_C^{d/2-1}$.

Using the standard tools [21] we convert the sum over KK modes into a contour integral. The divergent part is of the form $f_C(C_1 + a_\pi^2 C_2)$ and can be absorbed into renormalization of the tree-level Kähler potential. The remaining finite part is given by

$$\begin{aligned}\Delta\Omega &= \frac{4}{3M_p^2(4\pi)^2}\left(1 - \frac{2k\pi R}{3M_p^2}\Omega_0(Q_0)\right) k^2 a_\pi^2 \int_0^\infty dy y \log\left(1 - \frac{I_1(ya_\pi)K_1(y)}{K_1(ya_\pi)I_1(y)}\right) \\ a_\pi^2 &= e^{-k\pi R(T+\overline{T})} \frac{1 + \frac{2k\pi R}{3M_p^2}\Omega_\pi(Q_\pi)}{1 - \frac{2k\pi R}{3M_p^2}\Omega_0(Q_0)}\,.\end{aligned} \tag{4.9}$$

The Kähler potential $\Omega + \Delta\Omega$ contains all information about the contact terms between the hidden and observable sectors. In the limit of large warping, $a_\pi \to 0$ we can approximate $\Delta\Omega$ by:

$$\Delta\Omega \approx -\frac{4ck^2}{3M_p^2(4\pi)^2} e^{-2k\pi R(T+\overline{T})} \frac{\left(1 + \frac{2k\pi R}{3M_p^2}\Omega_\pi(Q_\pi)\right)^2}{1 - \frac{2k\pi R}{3M_p^2}\Omega_0(Q_0)} + \mathcal{O}(a_\pi^6 \log a_\pi)\,, \tag{4.10}$$

where $c = \int_0^\infty dy y^3 \frac{K_1(y)}{2I_1(y)} \approx 1.165$. On the other hand in the flat limit $k \to 0$ we find

$$\Delta\Omega = -\frac{16\zeta(3)}{3(4\pi)^2} \frac{1}{(2\pi R M_p)^2} \frac{1}{\left(T + \overline{T} - \frac{2}{3M_p^2}\Omega_0(Q_0) - \frac{2}{3M_p^2}\Omega_\pi(Q_\pi)\right)^2}\,. \tag{4.11}$$

5. On ambiguity in one-loop Kähler potential

We now compare the result of our computation to the previous works on the subject [3, 5, 4]. For simplicity, we restrict to the flat limit $k \to 0$. Expanding the $\Delta\Omega$ in eq. (4.11) in powers of Ω_i we obtain

$$\begin{aligned}\Delta\Omega = &-\frac{16\zeta(3)}{3(4\pi)^2} \frac{1}{(2\pi R M_p)^2} \left[\frac{1}{(T+\overline{T})^2} + \frac{4}{3M_p^2(T+\overline{T})^3}(\Omega_0(Q_0) + \Omega_\pi(Q_\pi)) + \right.\\ &\left. + \frac{4}{3M_p^4(T+\overline{T})^4}(\Omega_0(Q_0) + \Omega_\pi(Q_\pi))^2 + \cdots\right]. \end{aligned}\tag{5.1}$$

The first term describes the Casimir energy [21], the second corresponds to radion mediation [3] and the last one to brane-to-brane mediation of supersymmetry breaking [5]. At this order, all the terms in eq. (5.1) are the same as those derived in the literature. However the full formula eq. (4.11) is clearly different than that in ref. [5] and the discrepancy enters at the cubic order in Ω_i. As long as the brane sources are perturbative, the physical consequences of both formulations (summarized in eq. (5.1)) are the same. However when the brane sources are large (for $\Omega_i > 2\pi R\phi M_5^3$) conclusions derived in both formalisms may be completely different. In particular, from eq. (4.11) it is evident that a constant terms in the boundary Kähler potential, $\Omega_i = L + \ldots$, is equivalent to shifting T by a constant and therefore has no physical significance. This is certainly different than in ref. [5] where a large value of L was needed for obtaining positive soft mass terms.

The origin of this incompatibility can be traced to the different formulation of the 5d brane-world theory. The technical issue that affects the one-loop computation is the fact that in the off-shell formulation Ω_0 and Ω_π multiply brane kinetic terms of the gravity multiplet. These couplings remain after integrating out the auxiliary fields. On the other hand, in our purely on-shell Noether formulation such terms are absent. In order to understand this difference better, below we discuss supersymmetrization of a model with a brane Einstein-Hilbert term by means of the Noether procedure. For simplicity we restrict to the case where no brane matter is present. Thus we start with the brane lagrangian of the form

$$\mathcal{L} = e_4 M_5^3 L\delta(x_5)\left[\frac{1}{2}R_4 - i\overline{\psi}_\mu^+\gamma^{\mu\nu\rho}D_\nu\psi_\rho^+\right]. \tag{5.2}$$

In ordinary 4d supergravity this lagrangian would be supersymmetric up to four-fermion terms. But here $\delta\psi_\mu \sim \mathcal{F}_{\mu 5}\epsilon$ and so the variation of eq. (5.2) is non-zero already at the two-fermion level. To cancel it one has to add new zero- and two-fermion terms to eq. (5.2) as well as modify the supersymmetry transformation of the gravitino by terms proportional to $L\delta$. However once one arrives at a lagrangian in which all variations of order $L\delta$ cancel one finds that there are $L^2\delta^2$ variations that do not cancel. Therefore the Noether procedure must be continued and new singular terms of order $L^2\delta^2$ have to be added to eq. (5.2) to make the lagrangian supersymmetric at this order.[5] The story does not end at order δ^2. In order to maintain supersymmetry singular terms with higher and higher powers of δ are needed. However one can notice that there is a certain pattern emerging. It turns out that all the terms obtained by the Noether procedure can be obtained from the bulk action

$$\mathcal{L} = e_5 M_5^3 \left[\frac{1}{2}R_5 - i\overline{\psi}_\alpha \Gamma^{\alpha\beta\gamma} D_\beta \psi_\gamma - \frac{1}{4}\mathcal{F}_{\alpha\beta}\mathcal{F}^{\alpha\beta} + \cdots\right] \tag{5.3}$$

by a formal, singular redefinition of the $\phi \equiv e_5^5$ component of the 5d vielbein,

$$\phi \to \phi + L\delta(x_5)\,. \tag{5.4}$$

In addition, one should assume that only positive parity fields multiplied by $\delta(x_5)$ survive in the brane action.

For example by Noether procedure we get a series of graviphoton brane kinetic terms

$$\begin{aligned}\mathcal{L} &= \frac{1}{2\phi}M_5^3 e_4 \left[\frac{L}{\phi}\delta(x_5) - \frac{L^2}{\phi^2}\delta(x_5)^2 + \frac{L^3}{\phi^3}\delta(x_5)^2 + \cdots\right]\mathcal{F}_{\mu 5}^2 \\ &= \frac{1}{2}\left(\frac{1}{\phi + L\delta(x_5)} - \frac{1}{\phi}\right) M_5^3 e_4 \mathcal{F}_{\mu 5}^2\,.\end{aligned} \tag{5.5}$$

One can argue that the extremely singular term like the one in eq. (5.5) is indeed needed for supersymmetry. Indeed, the graviton and gravitino KK spectrum with brane kinetic terms of eq. (5.2) is given by solutions of

$$\tan(\phi\pi R m_n) = -\frac{1}{2}m_n L\,. \tag{5.6}$$

For the graviphoton the equation of motion reads $\partial_5\left(\frac{\partial_5\mathcal{A}_\mu}{\phi+L\delta(x_5)}\right) + \phi m_n^2 \mathcal{A}_\mu = 0$ To cancel all δ's we should arrange that $\partial_5\mathcal{A}_\mu$ behaves as $\sim \phi + L\delta(x_5)$ near the boundary. Matching this boundary condition with the bulk solution of the equations of motion we get precisely the quantization condition eq. (5.6). Getting this spectrum for the graviphoton would be impossible without brane $\mathcal{F}^2$ term or with any decent non-singular $\mathcal{F}^2\delta$ term. We conclude that for supersymmetrizing a 5d model with a boundary Einstein-Hilbert term it is indeed necessary to include an infinite series of singular δ^n terms in the action.

[5] In the following we ignore the mathematical subtleties involved in multiplication of distribution and manipulate δ's as if they were ordinary c-numbers.

We can now infer the relation between the brane-world action obtained by integrating out auxiliary fields in the off-shell formulation and the one obtained by our Noether procedure. The two are connected by a singular change of variables

$$\phi \to \phi - \frac{1}{3M_5^3}\Omega_0(Q_0)\delta(x_5) - \frac{1}{3M_5^3}\Omega_\pi(Q_\pi)\delta(x_5 - \pi R)\,. \tag{5.7}$$

If the two formalisms are in fact equivalent up to a change of variables why the computation of loop corrections yields different results? The difference can be traced to ambiguity of defining the behavior of bulk fields near the δ sources. In the example discussed in this section, after the redefinition $\phi \to \phi + L\delta(x_5)$ we kept only positive Z_2 parity fields in the brane action. But more generally, this redefinition yields also boundary terms involving negative Z_2 parity fields, for example

$$\mathcal{L} = -ie_4 M_5^3 L\delta(x_5)\overline{\psi_\mu^-}\gamma^{\mu\nu\rho}D_\nu\psi_\rho^-\,. \tag{5.8}$$

Now, once positive Z_2 parity fields are present in the brane action (like in eq. (5.2)), by equations of motions negative Z_2 parity fields behave as $\psi^- \sim \epsilon(x_5)Lm_n\psi^+$ near the boundary. Then the boundary term (5.8) contributes at the order $L^3m_n^3$ to the KK mode equations, provided we define the distribution $\delta(x_5)\epsilon^2(x_5)$ to be non-vanishing. In consequence, (5.8) affects the KK spectrum at the order $(L/\pi R\phi)^3$ and, by eq. (4.7), also the one-loop Kähler potential at the same order. Although (5.8) is a local operator, the scale at which its effects become important is controlled by a non-local quantity - the compactification length $\pi R\phi$. The reason is that the boundary value of ψ^- is sensitive to the KK mass spectrum and the latter obviously depends on non-local physics.

Concluding, the change of variable eq. (5.7) defines in fact a class of brane-world actions, depending on what regularization scheme we adopt to resolve the brane singularity. Physical predictions within this class of theories may differ at the third order in brane sources. As long as the brane sources are perturbative, the relevant physical quantities (e.g. soft mass terms) derived in both formulations are the same. However, if the brane sources are large (in the above example, if L is bigger that the compactification length $2\pi R\phi$) the low-energy observables may depend on how the brane singularity is regularized.

6. Conclusions

In this paper we used the Noether method to construct 5d on-shell gauged supergravity coupled to chiral matter multiplets on the branes. This turned out to lead to a slightly different set-up than that derived from the more commonly used off-shell formulation. Certain singularities that appear after integrating out the auxiliary fields are absent in the purely on-shell Noether formulation. This is due to a different choice of the fundamental degrees of freedom in the 5d theory.

Furthermore, we showed that our on-shell set-up allows for a simple computation of one-loop corrections. Comparison of our results with previous works showed an ambiguity in computation of the one-loop Kähler potential. This ambiguity is associated with arbitrariness in resolving the singularities associated with infinitely thin, delta-type branes.

As long as the brane sources are small (the reference scale being $M_5^3 2\pi R\phi$) this ambiguity has negligible effects on the low-energy physics. However in certain 5d models large brane sources are essential. One well-known example is the Dvali-Gabadadze-Porratti [22] model in which gravity is localized on a 4d brane in a semi-infinite flat extra dimension. We conclude that there is a whole class of supersymmetric completions of the DGP model that yield different low-energy predictions. Another example are set-ups with a gravitino brane mass term, $\mathcal{L} \sim W\delta(x_5)\psi_\mu^T\gamma^{\mu\nu}\psi_\nu + \text{h.c.}$. The limit $W \to \infty$ is sometimes considered as being equivalent to the set-up with supersymmetry broken by boundary conditions. In such limit there is also a continuous family of regularizations that results in different physics at low energies.

We expect that the Noether method can be easily carried over to other brane-world models, for example to 6d supergravity with matter on a brane of co-dimension two. This offers an opportunity to construct more general brane-world actions and study its low-energy phenomenology without a necessity of going through the off-shell calculus.

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A. Notation and conventions

We use the mostly minus metric signature $(+,-,-,-)$. The index conventions are the following: 5d Einstein indices $\alpha, \beta, \gamma \ldots = 0 \ldots 3, 5$, 5d Lorentz indices $a, b, c \ldots = 0 \ldots 3, 5$, 4d Einstein indices $\mu, \nu, \rho \ldots = 0 \ldots 3$, 4d Lorentz indices $m, n, \ldots = 0 \ldots 3$.

The 5d vielbein is denoted by e_α^a and satisfies $e_\alpha^a e_\beta^b \eta_{ab} = g_{\alpha\beta}$. e_5 is the determinant of the 5d vielbein, while by e_4 we denote the determinant of the 4d vielbein induced at the boundary. Similarly, R_5 denotes the 5d Ricci scalar, while R_4 denotes the Ricci scalar constructed from the 4d vielbein induced at the boundary. The inverse vielbein e_a^α satisfies $e_a^\alpha e_\beta^a = \delta_\alpha^\beta$.

5d gamma matrices are denoted as Γ^a while 4d gamma matrices are denoted as γ^m. They satisfy $\{\Gamma^a, \Gamma^b\} = 2\eta^{ab}$ and $\{\gamma^m, \gamma^n\} = 2\eta^{mn}$. The connection between the two sets is given by $\Gamma^m = \gamma^m$, $\Gamma^5 = i\gamma^5$. Furthermore $\Gamma^\alpha = e_a^\alpha \Gamma^a$, $\gamma^\mu = e_m^\mu \gamma^m$. The convention for γ^5 is $\gamma^5 = \text{diag}(-1,-1,1,1)$ and the chirality projection operators are $P_L = (1-\gamma^5)/2$, $P_R = (1+\gamma^5)/2$. All the fermions in 5d and 4d are in four-component Dirac notation (we don't use symplectic Majorana spinors). The 4d charge conjugation matrix $C = i\gamma^0\gamma^2\gamma^5$ satisfies $C^{-1} = C^T = C^\dagger = -C$, $C\gamma^m C^{-1} = (\gamma^m)^T$.

B. Brane action with a superpotential

One can extend the set-up studied in section 2 to include a non-trivial superpotential $W_0(Q_0)$ for the brane multiplet. We start with the case of flat supergravity in the bulk. In addition to those of eq. (2.3), the terms present in the globally supersymmetric limit are the following

$$\mathcal{L}_3 = e_4\delta(x_5)\left[-\left|\frac{\partial W_0}{\partial Q_0}\right|^2 + \frac{1}{2}\frac{\partial^2 W_0}{\partial Q_0^2}\psi_Q^T C\psi_Q - \frac{1}{2}\frac{\partial^2 \overline{W_0}}{\partial Q_0^{\dagger 2}}\overline{\psi}_Q C\overline{\psi}_Q{}^T\right]. \tag{B.1}$$

The supersymmetry transformation of the chiral fermion is supplemented by $\delta\psi_Q = \frac{1}{\sqrt{2}} \times \frac{\partial\overline{W_0}}{\partial Q_0^\dagger} C\overline{\epsilon}^T$. In the presence of the superpotential the matter supercurrent is modified. The gravitino couples additionally as

$$\mathcal{L}_4 = -\frac{i}{\sqrt{2}}e_4\delta(x_5)\frac{\partial W_0}{\partial Q_0}\psi_Q^T C\gamma^\mu\psi_\mu + \text{h.c.} \tag{B.2}$$

Besides, up to four-fermion terms local supersymmetry requires one more term

$$\mathcal{L}_5 = e_4\delta(x_5)\left[-\frac{1}{2}W_0\overline{\psi}_\mu\gamma^{\mu\nu}C\overline{\psi}_\nu{}^T + \text{h.c.}\right]. \tag{B.3}$$

Furthermore one modifies the trasformation law of ψ_5 by

$$\delta\psi_5 = -\delta(x_5)W_0 C\overline{\epsilon}^T. \tag{B.4}$$

The action on the other brane is analogous with $\delta(x_5) \to \delta(x_5 - \pi R)$, $W_0(Q_0) \to W_\pi(Q_\pi)$. Again, no singular terms arise in this construction. One important comment is in order here. The gravitino brane mass term acts as a source for negative Z_2 parity gravitino so that $M_5^3\psi_\mu^- \sim \epsilon(x_5)W_0 C\overline{\psi_\mu^+}^T$ near the boundary. Thus, in general, ψ_μ^- can also couple to the brane matter. Therefore we have to reconsider the question which combination of the two bulk gravitinos should couple to the matter supercurrent in eq. (2.5) and eq. (B.2). It turns out that the answer depends on how the delta singularity is regularized. But whatever regularization we choose there is always one combination of the two bulk gravitinos $\epsilon(x_5)\psi_\mu^+\sin\theta + C\overline{\psi_\mu^-}^T\cos\theta$ that vanishes at the brane in the limit when the regulator is removed. Then the orthogonal combination $\psi_\mu = \psi_\mu^+\cos\theta - \epsilon(x_5)C\overline{\psi_\mu^-}^T\cos\theta$ couples to the brane matter. The angle of rotation is given by $\theta = W_0/(2M_5^3) + \mathcal{O}(W_0^3)$, where the higher order terms in W_0 are regularization dependent. Coupling this combination ψ_μ to the matter supercurrent yields also correct (consistent with the general supergravity action [20]) couplings in the 4d effective action, up to $\mathcal{O}(W_0^3)$ terms. But another conclusion from this discussion is that the low-energy 4d supergravity is regularization independent only up to terms cubic in the brane superpotential W_0.

A careful treatment of the boundary conditions for gravitinos is even more important when warped supergravity ($k \neq 0$) is present in the bulk. Ref. [18] discussed this problem for the case when brane matter is absent and W_0 is a constant. It was found that the brane action has to be modified already at the purely bosonic level. The brane tension term is given by:

$$\mathcal{L} = -6M_5^3 k e_4 \delta(x_5) \frac{1 - \frac{W_0^2}{4M_5^6}}{1 + \frac{W_0^2}{4M_5^6}} \approx -6M_5^3 k e_4 \delta(x_5)(1 - \frac{W_0^2}{2M_5^6}) + \mathcal{O}(W_0^4)\,. \tag{B.5}$$

There is a similar term on the other brane with $k \to -k$, $W_0 \to W_\pi$. Therefore, in the presence of gravitino brane mass terms the Randall-Sundrum tuning between the bulk cosmological constant and the brane tension is lost. The background solution is then of the Randall-Karch type [23] with AdS_4 symmetry of the 4d spacetime. Note that is consistent with what we obtain in the low energy 4d supergravity description. With the Kähler potential eq. (3.8) and the superpotential eq. (3.10) we obtain the scalar potential $V = \frac{6k\pi R}{f_E(\phi)^2 M_p^2}(|W_0|^2 - e^{-4k\pi R\phi}|W_\pi|^2)$ which is what follows from the warped compactification with the brane tension of eq. (B.5).

The case when warped supergravity, brane matter and brane superpotentials are present simultaneously is technically more involved and will be left for future studies.

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